Matter-Wave Interferometry Vibration Isolation

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3 October, 1990

ONR
N00014-90-J-1475

In this note we provide a brief summary of vibration isolation techniques and their application to our neutral atom interferometry experiments, at the UC Berkeley Physics Department. Naturally, the difficulty in achieving acceptable vibration isolation for any given experiment depends largely upon the noise background of the laboratory, the noise generated by the experimental apparatus itself, as well as the tolerable noise sensitivity of the experiment. Since neutral atom interferometers may be configured to act as ultra-sensitive inertial sensors, their vibrational noise sensitivity is inherently high.

1. What constitutes signal & what constitutes noise?

The initial experiments at UCB are to simply demonstrate neutral atom interference. For these experiments any deviations from an inertial reference frame for the apparatus represent a potential noise source. Even the quasi-constant earth's gravitational field and rotation can be considered as very low frequency noise components, although subsequent experiments will consider these as known test signal's to be measured.

2. Vibration sensitivity of neutral atom interferometry experiments:

Our proposed neutral atom interferometer includes the following components: (a) a source of slow, cold atoms, (b) a sequence of transmission diffraction gratings, and (c) an atomic particle detector. The interferometer's parameters were selected to de-emphasize its inertial sensitivity and thereby assure success of the initial experiments. Nonetheless, it is still quite sensitive to inertial forces, such as those caused by vibrationally induced acceleration. Its sensitive axis is in a direction perpendicular to the source-detector axis and perpendicular to the grating slits' long direction. It has negligible sensitivity to inertial forces acting perpendicular to

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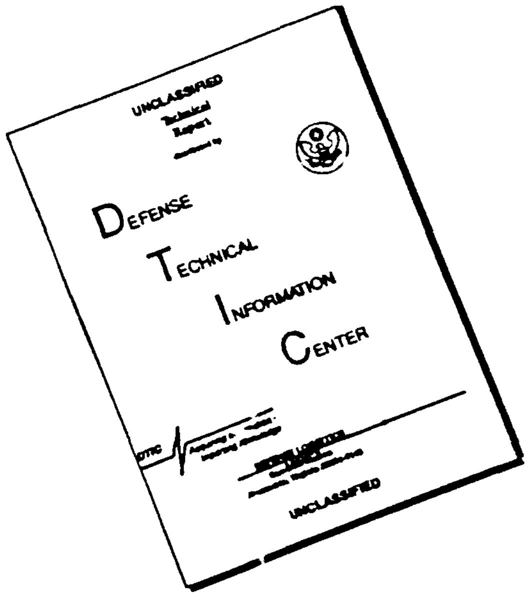

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its sensitive axis.

Unfortunately, for any experiment in a terrestrial laboratory, gravity cannot be eliminated. Thus, it is worthwhile to orient the apparatus so that gravity is perpendicular to the sensitive axis. In our case, this is done by having the beam propagate vertically. An advantage of this orientation is that the axial alignment remains independent of atomic velocity.

The purpose of our initial experiments is simply to detect interference fringes. Hence, one desires that the peak-to-peak worst-case vibration noise be limited to provide less than one fringe shift in whatever observation period is necessary for positive identification and measurement of the atomic fringe structure. In simplest terms, this requirement translates to the requirement that the worst-case peak-to-peak vibration amplitude (relative to an inertial frame) of any grating be much less than one slit width of that grating. If phase sensitive detection is employed, this limiting amplitude constrains the apparatus allowed vibration only over the bandwidth of the fringe detector, which, in turn, can be made quite narrow, and furthermore, can be centered at a vibrationally quiet portion of the spectrum. With phase sensitive detection the above constraint may be relaxed at frequencies outside this bandwidth.

Why does the slit width represent a limiting amplitude for vibrations? Indeed, for periodic vibrations of constant amplitude the amplitude of the resulting acceleration scales with the square of the vibration frequency. Thus, one might expect that the interferometer fringe shift (proportional to linear acceleration) will scale similarly. Fortunately, this is not the case for periodic accelerations with frequencies higher than the inverse transit time of atoms through the interferometer. For such frequencies the accelerational sensitivity decreases inversely with frequency squared, so that the limiting spatial amplitude for vibrations is still just the slit width.

To visualize this dependence, consider in an inertial frame waves passing through a set of vibrating gratings. The diffraction pattern at the final grating (and the Moiré pattern formed by this pattern and the final grating) is given by the Kirchoff diffraction integral over possible paths (in the inertial frame)

from the source, through all open slits to the final grating. The possible paths traversed by any given wavefront constitute those open at the time of its passage. Thus, even though a grating may rapidly vibrate during the passage of a wavefront through a slit, so long as the majority of paths offered by open slits remain open for the passage of subsequent wavefronts, then the Kirchoff diffraction integral will be neglegibly altered. That is, as long as a only negligible fraction of each slits' open cross-section is affected by the vibration, the diffraction pattern will be maintained. This will be true as long as the worst-case wiggling of the edges of these paths remains small with respect to a slit width.

Our initial experiments anticipate the use of about 1/2 to 1 micron slits, a path length of 0.32m, and a lowest velocity (with correspondingly highest accelerational sensitivity) of 5 - 10 m/sec. The worst case vibrational noise occurs at a frequency of $1/\tau$(transit), or 5 - 10 Hz. Since externally produced high frequency (>> 1Hz) vibrations are comparatively easy to isolate from the apparatus but the support structure must pass zero frequency, it is the lowest frequency components (0.1 - 5 Hz) that are potentially the most troublesome.

Another potential source of noise is that due to structural flexure within the apparatus. Such flexure can allow one grating to vibrate relative to another one and thereby couple additional noise into the system. Unless sufficient damping is provided, apparatus generated noise may be trapped within the isolated apparatus. Structural flexure resonances can then cause amplification of these vibrations and significant relative motion of the gratings will produce additional noise. Fortunately, relative motion of the gratings is detectable with *in-situ* optical interferometry and, if found present, can be remedied by eliminating resonances and/or introduction of additional damping.

3. Noise sources in Room 318 LeConte, UCB Physics Dept.:

Potential external sources of vibration include various forms of cultural noise (e.g. hallway traffic), building plant noise (typically rotating machinery), seismic activity, etc. Its magnitude depends on the laboratory construction, location within

the building and the time of day. On the third floor of LeConte Hall, all of these sources have been measured at various times, with frequency spectra in the range of a few Hz to a few 10's of Hz. Vibrations of the same order of magnitude are measurable in vertical and horizontal directions, as well as in rolling motions of the floor. Typical vibration amplitudes in Room 318 are of order 1-4 microns. Although the floor's rolling motion is large, suspending the entire apparatus on a two-axis knife-edge bearing prevents coupling this motion into the rotational modes of the apparatus. With significant apparatus height above the floor, the rolling motion produces an amplified horizontal motion of the apparatus. The rolling motion thus requires significant horizontal isolation of the apparatus center of gravity, provided by a flexible leg support structure and damped pneumatic pistons. Isolation ratios of 10 to 100 from floor vibrations will suffice, even for experiments not using phase sensitive detection. Phase sensitive detection can further reduce vibrational noise to total insignificance.

4. Techniques:

There are two basic popular methods for isolation of scientific apparatus: active and passive. Passive (conventional) isolation systems are based on the low-pass filter action of a spring-mass-dashpot linear system. Higher isolation using the same principles is available by cascading such filters (as is commonly done in gravitational wave detection experiments). The basic physics of such isolation is given in the attached excerpt from a Newport Research Corporation catalog. Passive isolation systems and components are commercially available for supporting large apparatus. Unfortunately, such commercial components are awkward to use with an apparatus with significant vertical height (such as ours).

Active isolation systems sense vibrational acceleration of the apparatus with an accelerometer and apply a corrective force via an electronic feedback system. Such systems are complex and costly. Commercial active systems are presently available only for small apparatus.

The present system at UCB is passive and successfully

isolates building noise to the required degree not to require phase sensitive detection. Apparatus self-noise at present dominates. It is evidently due to vibrations caused by boiling liquids in the diffusion pumps and liquid nitrogen traps. Significant noise is found to exist in the isolator normal modes only when the pumps are on and the traps are full. Experiments currently underway will determine whether this noise can be brought to an acceptable level by damping improvements. If not, these pumps and traps may be replaced with sorption roughing pumps and ion high-vacuum pumps.

Fundamentals of Vibration

Many problems of vibration are caused by structural resonances of the measurement apparatus — for example the table on which an optical interferometry experiment is performed. Vibration and vibration isolation are both intimately connected with the phenomenon of resonance, which is illustrated in this section by the two basic models below.

Model I: The Simple Harmonic Oscillator

The simple harmonic oscillator consists of a rigid mass M connected to an ideal linear spring as shown in Figure A.

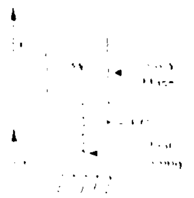

Fig. A: Simple Harmonic Oscillator described by Mẍ = −x/C

The spring has a static compliance C such that the change in length of the spring Δx that occurs in response to a force F is

$$\Delta x = CF$$

Note that the compliance C is the inverse of the spring stiffness, denoted by k, such that k = 1/C.

If the spring mass system is driven by a sinusoidal displacement with frequency ω and peak amplitude |u|, it will produce a sinusoidal displacement of the mass M with peak amplitude |x| at the same frequency ω. The steady state ratio of the amplitude of the mass motion |x| to the spring end motion |u| is called the transmissibility T and is given by

$$T = \frac{|x|}{|u|} = \frac{1}{1 - \frac{\omega^2}{\omega_0^2}}$$

where ω_0 is the resonance or natural frequency of the system given by

$$\omega_0 = \sqrt{\frac{1}{CM}}$$

Note that the natural frequency of the system, ω_0, is determined solely by the mass and the spring compliance. It decreases for a larger mass or a more compliant (softer) spring. The transmissibility T of the system is plotted as a function of the ratio ω/ω_0 on a log-log plot in Figure B.

The three characteristic features of this system are:

1) For $\omega \ll \omega_0$, well below the resonance frequency, the transmissibility $T \approx 1$, so the motion of the mass is the same as the motion of the other end of the spring.

2) For $\omega \approx \omega_0$, near resonance, the motion of the spring end is amplified, and the motion of the mass |x| is greater than that of |u|. For an undamped system, the motion of the mass becomes theoretically infinite for $\omega = \omega_0$.

3) For $\omega \gg \omega_0$, the resulting displacement |x| decreases in proportion to $1/\omega^2$. In this case, the displacement |u| applied to the system is not transmitted to the mass. In other words, the spring acts like an isolator.

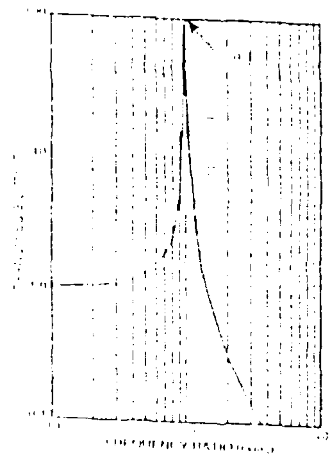

Fig. B: Transmissibility of a simple harmonic oscillator

Model II: The Damped Simple Harmonic Oscillator

In the first model, we considered an undamped system in which there is no mechanism to dissipate mechanical energy from the mass-spring system. Damping refers to a mechanism that removes the mechanical energy from the system — very often as heat. A damped simple harmonic oscillator is shown schematically in Figure C.

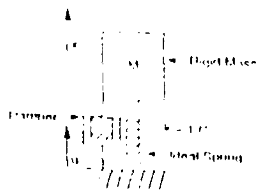

Fig. C: Damped Simple Harmonic Oscillator described by Mẍ + bẋ + (1/C)x = 0

A rigidly connected damper is expressed mathematically by adding a damping term proportional to the velocity of the mass and to the differential equation describing the motion. For an external force that results in a displacement amplitude |u| of the end of the spring as in Model I, the transmissibility T of the damped system becomes

$$T = \frac{|x|}{|u|} = \sqrt{\frac{1 + \left(2\xi\frac{\omega}{\omega_0}\right)^2}{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \left(2\xi\frac{\omega}{\omega_0}\right)^2}}$$

where ξ is a "damping" coefficient given by

$$\xi = \frac{b}{2\sqrt{\frac{M}{C}}}$$

A plot of the transmissibility T is shown in Figure D for various values of the damping coefficient ξ. In the limit where ξ approaches zero, the curve becomes exactly the same as in Model I, that is, there is infinite amplification at the resonance frequency ω_0. As the damping increases, the amplitude at resonance decreases. However, the "roll off" at higher frequencies decreases (i.e. the transmissibility declines more slowly as damping increases). For $\omega/\omega_0 \gg 1$, note that the motion of |x| is proportional to $1/\omega$, as compared to Model I where at high frequencies the motion of |x| decreases as $1/\omega^2$.

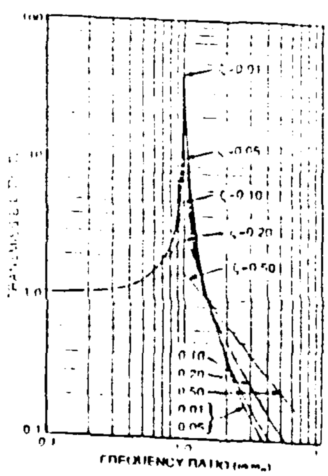

Fig. D: Transmissibility of a damped oscillating system with various values of damping coefficient (ξ)

Pneumatic isolators are one of the best methods of vibration isolation in critical applications. When properly designed and carefully constructed, their performance combines the "fast roll off" of the simple harmonic oscillator at vibration frequencies above resonance with the "low amplification at resonance" of the damped harmonic oscillator near resonance.

The basic design for a pneumatic isolator with damping is shown in Figure I. The isolated mass M (for example, an optical table, or precision instrument such as a microscope) is supported by a piston which rests on a flexible rolling diaphragm. The diaphragm separates the piston from the top section of the air chamber called the "spring chamber." In damped systems, air can flow between the spring chamber and a secondary chamber, called the "damping chamber" through a flow restriction, usually a small orifice. As air flows through the orifice, energy is dissipated, reducing the amplification of the isolator at resonance. The theoretical analysis of this system with damping is significantly different from that of a simple mass spring system, and the resulting ratio of the displacement of the isolated mass to the displacement of the floor is given by:

$$T = \left|\frac{x}{u}\right| = \sqrt{\frac{1 + D\left(2\zeta\frac{\omega}{\omega_n}\right)^2}{\left(1 - B\frac{\omega^2}{\omega_n^2}\right)^2 + D\left(2\zeta\frac{\omega}{\omega_n}\right)^2\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2}}$$

where:

$$\omega_n = \sqrt{\frac{gA}{V_s}\left(1 + \frac{P_g}{P_a}\right)}, \text{ and}$$

ω_n is the natural frequency of the undamped system

ζ is the damping coefficient determined by the details of the damping mechanism

ω is the frequency of the vibration

B, D are constants which depend on the details of the isolator design

A is the cross sectional area of the piston

V_s is the spring chamber volume

P_g is the gauge pressure, that is the pressure in the isolator (above the pressure outside) which is dependent on the mass supported by the leg

P_a is the atmospheric pressure

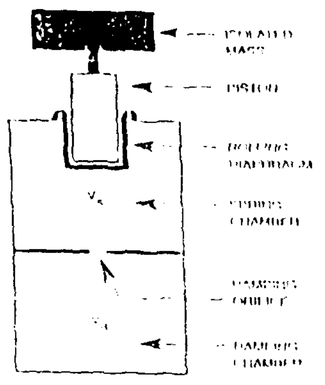

Fig. I The pneumatic isolator with damping

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Note that the transmissibility of a damped, pneumatic isolator is very different from that of the damped simple harmonic oscillator of Model II, and illustrates the main features of pneumatic isolators:

1) **High performance over a wide range of loads.** The resonance frequency ω_n of the isolator is only weakly dependent upon the load M.

2) **Low natural frequency when the system is operated at several times atmospheric pressure.** In practice Newport pneumatic isolators are operated so that the air pressure in each leg is between about 30 and 80 psi.

3) **$1/\omega^2$ "roll off" even with high damping and low resonance peak transmissibility.** The steep decrease in transmissibility T as the frequency increases is much faster in pneumatic isolators than the damped simple harmonic oscillator of Model II, for which the transmissibility decreases as $1/\omega$ at high frequencies. Effectively, the damping is present only near resonance, where it is most needed.

Newport has found that the transmissibility T and resonant frequency ω_n can differ considerably from theory due primarily to effects in the design of the diaphragm and subtleties in engineering the piston and chamber design. The actual measurement of the vertical transmissibility of Newport's XL-A series leg system, is shown in Figure F.

Designing Effective Horizontal Isolation

The pneumatic isolation described above provides isolation primarily from vertical vibrations only. (Actually, the diaphragm alone provides some horizontal isolation.) For improved, high performance horizontal vibration isolation another technique must be used. Newport uses a patented damped pendulum design which is schematically shown in Figure G.

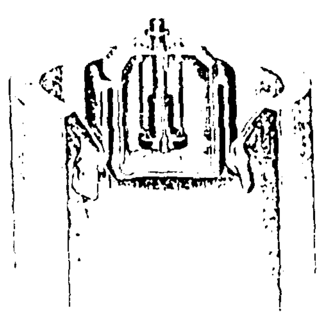

Fig. G Newport's patented horizontal isolation action

As the floor moves relative to the isolated object, the table behaves similar to a pendulum with the pivot point moving back and forth. The equations of motion thus would be the same as those of a simple harmonic oscillator, and the natural frequency of this system is:

$$f_n = \frac{\omega_n}{2\pi} = \frac{\sqrt{g/L}}{2\pi}$$

where g is the acceleration of gravity, and L is the length of the pendulum.

The actual system dynamics are more complex, and the measured natural frequency of Newport's horizontal isolation is about 1.8 Hz. For frequencies near resonance, there is amplification. However, the amount of amplification is determined by the amount of damping in the system. (Newport's XL-Series isolators have been optimally damped both vertically and horizontally.)

An important feature of Newport's patented horizontal isolation technique is that horizontal vibrations are not coupled into vertical vibrations to achieve damping, as is the case with "gimballed pistons." Actual measured transmissibility data (Figure H) shows the horizontal isolation of the pendulum.

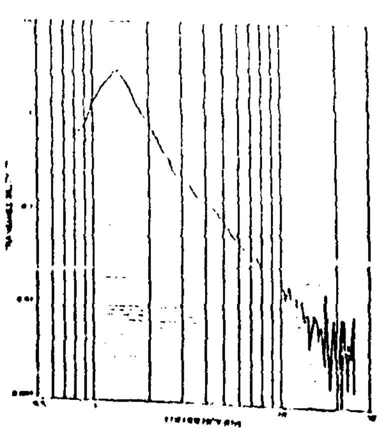

Fig. F Measured vertical transmissibility of XL-A Series pneumatic isolator

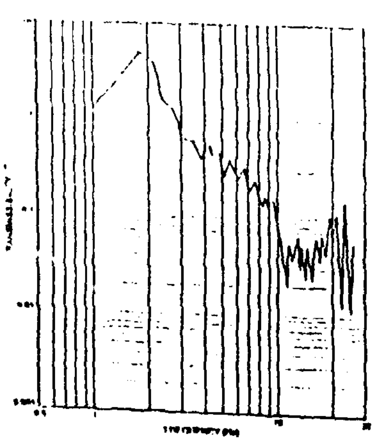

Fig. H Measured horizontal transmissibility of XL-A Series